APPENDIX G

DERIVATION OF EQUATIONS FOR CRITICAL SLIP-PLANE ANGLE (α) FOR DRIVING AND RESISTING WEDGES

G-1. Derivation of Critical Slip-Plane Angle (α) for Driving Wedges for the Static Condition. In the following paragraphs the equations for the critical slip-plane angle (α) will be derived for the static case for a driving wedge.

a. Introduction. The wedge equation (Equation 3-23) with the uplift term omitted is:

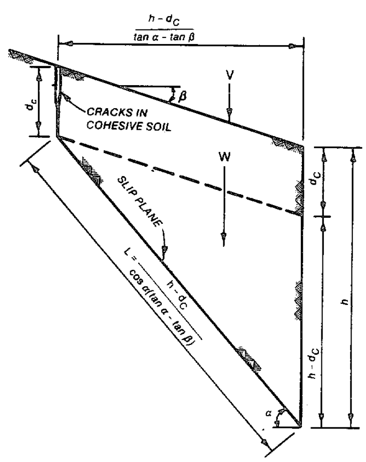
$$P_{EE} = \frac{(W + V)(1 - \tan\phi \cot\alpha)\tan\alpha}{1 + \tan\phi\tan\alpha} - \frac{cL}{\cos\alpha(1 + \tan\phi\tan\alpha)} \qquad [G-1]$$

where

$$L = \text{length of slip plane} = \frac{h - d_c}{\cos\alpha\,(\tan\alpha - \tan\beta)}$$

$$W = \frac{\gamma\left(h^2 - d_c^2\right)}{2\,(\tan\alpha - \tan\beta)}$$

as shown in the figure below.



W = WEIGHT OF SOIL IN WEDGE,

V = ANY SURFACE LOAD OTHER THAN A UNIFORM SURCHARGE

Inserting the terms for L and W into Equation G-1 yields,

$$P_{EE} = \frac{\gamma(h^2 - d_c^2)}{2\ (\tan \alpha - \tan \beta)} \cdot \frac{(1 - \tan \phi \cot \alpha\ ) \tan \alpha}{1 + \tan \phi \tan \alpha}$$

$$+ \frac{V(1 - \tan \phi \cot \alpha) \tan \alpha}{1 + \tan \phi \tan \alpha} - \frac{c(h - d_c)}{\cos^2 \alpha\ (\tan \alpha - \tan \beta)(1 + \tan \phi \tan \alpha)}$$

Using the trigonometric identity $\frac{1}{\cos^2 \alpha} = \sec^2 \alpha = 1 + \tan^2 \alpha$ and putting all terms over a common denominator

$$P_{EE} = \left[\gamma\left(h^2 - d_c^2\right)(\tan \alpha - \tan \phi) + 2V\ (\tan \alpha - \tan \phi)(\tan \alpha - \tan \beta)\right.$$

$$\left. - 2c(h - d_c)(1 + \tan^2 \alpha)\right] \div \left[2\ (\tan \alpha - \tan \beta)(1 + \tan \phi \tan \alpha)\right] \qquad [G\text{-}2]$$

The first derivative of P_{EE} with respect to α may now be taken, and set equal to zero. The resulting expression is solved for the critical value of $\tan \alpha$. For simplification, the calculus is done in parts.

b. First Derivative of γ Term. Grouping the terms from Equation G-2 that are associated with γ yields,

$$P_{EE\gamma} = \frac{\gamma\left(h^2 - d_c^2\right)}{2} \circ \frac{\tan \alpha - \tan \phi}{(\tan \alpha - \tan \beta)(1 + \tan \phi \tan \alpha)}$$

$$P_{EE\gamma} = \frac{\gamma\left(h^2 - d_c^2\right)}{2} \circ \frac{\tan \alpha - \tan \phi}{\tan \phi \tan^2 \alpha + (1 - \tan \phi \tan \beta) \tan \alpha - \tan \beta} = \frac{m_\gamma}{n} \qquad [G\text{-}3]$$

The first derivative of Equation G-3 is

$$\frac{dP_{EE\gamma}}{d\alpha} = \frac{\frac{ndm_\gamma}{d\alpha} - \frac{m_\gamma dn}{d\alpha}}{n^2}$$

The denominator may be neglected because it is the same for all terms in Equation G-2.

$$\frac{dm_\gamma}{d\alpha} = a \sec^2 \alpha \text{ , let } \frac{\gamma\left(h^2 - d_c^2\right)}{2} = a$$

$$\frac{dn}{d\alpha} = \left[2 \tan \phi \tan \alpha + (1 - \tan \phi \tan \beta)\right] \sec^2 \alpha$$

$$\frac{ndm_\gamma}{d\alpha} = a\left[\tan \phi \tan^2 \alpha + (1 - \tan \phi \tan \beta) \tan \alpha - \tan \beta\right] \sec^2 \alpha$$

$$\frac{m_\gamma dn}{d\alpha} = a\Big[2 \tan \phi \tan^2 \alpha - (2 \tan^2 \phi + 1 - \tan \phi \tan \beta) \tan \alpha$$

$$+ \tan^2 \phi \tan \beta - \tan \phi(1 - \tan \phi \tan \beta)\Big] \sec^2 \alpha$$

$$\frac{ndm_\gamma}{d\alpha} - \frac{m_\gamma dn}{d\alpha} = \frac{\gamma\left(h^2 - d_c^2\right)}{2}\Big[-\tan \phi \tan^2 \alpha + 2 \tan^2 \phi \tan \alpha$$

$$+ \tan \phi(1 - \tan \phi \tan \beta) - \tan \beta\Big] \sec^2 \alpha \qquad \text{[G-4]}$$

c. First Derivative of c Term. Grouping the terms from Equation G-2 that are associated with c yields,

$$P_{EEc} = -c(h - d_c) \cdot \frac{1 + \tan^2 \alpha}{\tan \phi \tan^2 \alpha + (1 - \tan \phi \tan \beta) \tan \alpha - \tan \beta} = \frac{m_c}{n}$$

Let $-c(h - d_c) = a$

$$\frac{dm_c}{d\alpha} = a(2 \tan \alpha) \sec^2 \alpha$$

$$\frac{dn}{d\alpha} = \left[2 \tan \phi \tan \alpha + (1 - \tan \phi \tan \beta)\right] \sec^2 \alpha$$

$$\frac{ndm_c}{d\alpha} = a\left[2 \tan \phi \tan^3 \alpha + 2(1 - \tan \phi \tan \beta) \tan^2 \alpha - 2 \tan \beta \tan \alpha\right] \sec^2 \alpha$$

$$\frac{m_c dn}{d\alpha} = a\left[2 \tan \phi \tan^3 \alpha + (1 - \tan \phi \tan \beta) \tan^2 \alpha + 2 \tan \phi \tan \alpha + (1 - \tan \phi \tan \beta)\right] \sec^2 \alpha$$

$$\frac{ndm_c}{d\alpha} - \frac{m_c dn}{d\alpha} = -c(h - d_c)\left[(1 - \tan \phi \tan \beta) \tan^2 \alpha - 2 (\tan \phi + \tan \beta) \tan \alpha - (1 - \tan \phi \tan \beta)\right] \sec^2 \alpha \qquad \text{[G-5]}$$

d. <u>First Derivative of V Term</u>. Grouping the terms from Equation G-2 that are associated with the V term yields,

$$P_{EEV} = \frac{V\,[\tan^2 \alpha - (\tan \phi + \tan \beta) \tan \alpha + \tan \phi \tan \beta]}{\tan \phi \tan^2 \alpha + (1 - \tan \phi \tan \beta) \tan \alpha - \tan \beta} = \frac{m_V}{n}$$

$$\frac{dm_V}{d\alpha} = V[2 \tan \alpha - (\tan \phi \tan \beta)] \sec^2 \alpha$$

$$\frac{dn}{d\alpha} = [2 \tan \phi \tan \alpha + (1 - \tan \phi \tan \beta)] \sec^2 \alpha$$

$$\frac{ndm_v}{d\alpha} = V\left\{\left[2 \tan\phi \tan^3\alpha + (2 - 3\tan\phi\tan\beta - \tan^2\phi)\right]\tan^2\alpha\right.$$

$$- \left[3\tan\beta + \tan\phi - \tan\phi\tan\beta(\tan\phi + \tan\beta)\right]\tan\alpha$$

$$\left. + \tan\beta(\tan\phi + \tan\beta)\right\}\sec^2\alpha$$

$$\frac{m_v dn}{d\alpha} = V\left\{\left[2\tan\phi\tan^3\alpha + (1 - 3\tan\phi\tan\beta - 2\tan^2\phi)\right]\tan^2\alpha\right.$$

$$+ \left[2\tan^2\phi\tan\beta - (1 - \tan\phi\tan\beta)\right]\tan\alpha$$

$$\left. + \tan\phi\tan\beta(1 - \tan\phi\tan\beta)\right\}\sec^2\alpha$$

$$\frac{ndm_v}{d\alpha} - \frac{m_v dn}{d\alpha} = V\left[(1 + \tan^2\phi)\tan^2\alpha - 2\tan\beta(1 + \tan^2\phi)\tan\alpha\right.$$

$$\left. + \tan^2\beta(1 + \tan^2\phi)\right]\sec^2\alpha \qquad [G-6]$$

e. Summation of Terms. From Equations G-4, G-5, and G-6, set the sum of

$$\left(\frac{ndm_\gamma}{d\alpha} - \frac{m_\gamma dn}{d\alpha}\right) + \left(\frac{ndm_c}{d\alpha} - \frac{m_c dn}{d\alpha}\right) + \left(\frac{ndm_v}{d\alpha} - \frac{m_v dn}{d\alpha}\right)$$

equal to zero, and divide by $\sec^2\alpha$.

$$\left[-\frac{\gamma\left(h^2 - d_c^2\right)\tan\phi}{2} - c(h - d_c)(1 - \tan\phi\tan\beta) + V(1 + \tan^2\phi)\right]\tan^2\alpha$$

$$+\left[\frac{\gamma\left(h^2 - d_c^2\right)(2\tan^2\phi)}{2} + 2c(h - d_c)\ (\tan\phi + \tan\beta)\right.$$

$$\left. - 2V\tan\beta(1 + \tan^2\phi)\right]\tan\alpha$$

$$+\left\{\frac{\gamma\left(h^2 - d_c^2\right)}{2}[\tan\phi(1 - \tan\phi\tan\beta) - \tan\beta]\right.$$

$$\left. + c(h - d_c)(1 - \tan\phi\tan\beta) + V\tan^2\beta(1 + \tan^2\phi)\right\} = 0$$

Divide all terms by $-\frac{\gamma(h^2 - d_c^2)}{2}$.

$$\left[\tan\phi + \frac{2c(1 - \tan\phi\tan\beta)}{\gamma(h + d_c)} - \frac{2V(1 + \tan^2\phi)}{\gamma\left(h^2 - d_c^2\right)}\right]\tan^2\alpha$$

$$-\left[2\tan^2\phi + \frac{4c(\tan\phi + \tan\beta)}{\gamma(h + d_c)} - \frac{4V\tan\beta(1 + \tan^2\phi)}{\gamma\left(h^2 - d_c^2\right)}\right]\tan\alpha$$

$$-\left[\tan\phi(1 - \tan\phi\tan\beta) - \tan\beta + \frac{2c(1 - \tan\phi\tan\beta)}{\gamma(h + d_c)}\right.$$

$$\left. + \frac{2V\tan^2\beta(1 + \tan^2\phi)}{\gamma\left(h^2 - d_c^2\right)}\right] = 0 \qquad \text{[G-7]}$$

In Equation G-7, let the coefficient of $\tan^2\alpha$ equal A, let the coefficient of $\tan\alpha$ equal $-Ac_1$, and let the third coefficient equal $-Ac_2$. Equation G-7 yields

$$A \tan^2 \alpha - Ac_1 \tan \alpha - Ac_2 = 0$$

$$\tan^2 \alpha - c_1 \tan \alpha - c_2 = 0$$

and

$$\tan \alpha_{crit.} = \frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}$$

or

$$\alpha_{crit.} = \tan^{-1} \left(\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2} \right) \qquad [G\text{-}8]$$

The coefficients c_1 and c_2 are defined as

$$c_1 = \frac{2 \tan^2 \phi + \dfrac{4c (\tan \phi + \tan \beta)}{\gamma(h + d_c)} - \dfrac{4V \tan \beta \left(1 + \tan^2 \phi\right)}{\gamma\left(h^2 - d_c^2\right)}}{A} \qquad [G\text{-}9]$$

$$c_2 = \frac{\tan \phi(1 - \tan \phi \tan \beta) - \tan \beta + \dfrac{2c(1 - \tan \phi \tan \beta)}{\gamma(h + d_c)} + \dfrac{2V \tan^2 \beta \ (1 + \tan^2 \phi)}{\gamma\left(h^2 - d_c^2\right)}}{A} \qquad [G\text{-}10]$$

where

$$A = \tan \phi + \frac{2c(1 - \tan \phi \tan \beta)}{\gamma(h + d_c)} - \frac{2V\left(1 + \tan^2 \phi\right)}{\gamma\left(h^2 - d_c^2\right)} \qquad [G\text{-}11]$$

G-2. Derivation of Equation for V_{max}. The equations for calculating the critical value of α are not valid when the value of V is too large. When V is equal to or greater than a certain maximum value (V_{max}) the value of α should be set in accordance with Figure 3-15. The value of V_{max} is found by setting Equation 3-28 equal to zero and solving.

$$\tan\phi + \frac{2c(1 - \tan\phi\tan\beta)}{\gamma(H + d_c)} - \frac{2V_{max}(1+\tan^2\phi)}{\gamma(h^2 - d_c^2)} = 0$$

$$\frac{2V_{max}(1+\tan^2\phi)}{\gamma(h^2 - d_c^2)} = \frac{\gamma(h + d_c)\tan\phi + 2c(1 - \tan\phi\tan\beta)}{\gamma(h - d_c)}$$

$$V_{max} = \frac{\gamma\left(h^2 - d_c^2\right)\tan\phi + 2c(h - d_c)(1 - \tan\phi\tan\beta)}{2(1 + \tan^2\phi)} \qquad \text{[G-12]}$$

G-3. Equations for Critical Slip-Plane Angles for Resisting Wedges for the Static Condition. The following equations, for resisting wedges, where derived from Equation 3-33 in the same manner used for deriving the driving wedge equations for the static condition.

$$A = \tan\phi + \frac{2c(1 + \tan\phi\tan\beta)}{\gamma h} + \frac{2V(1 + \tan^2\phi)}{\gamma h^2} \qquad \text{[G-13]}$$

$$c_1 = \frac{2\tan^2\phi + \dfrac{4c(\tan\phi - \tan\beta)}{\gamma h} - \dfrac{4V\tan\beta(1 + \tan^2\phi)}{\gamma h^2}}{A} \qquad \text{[G-14]}$$

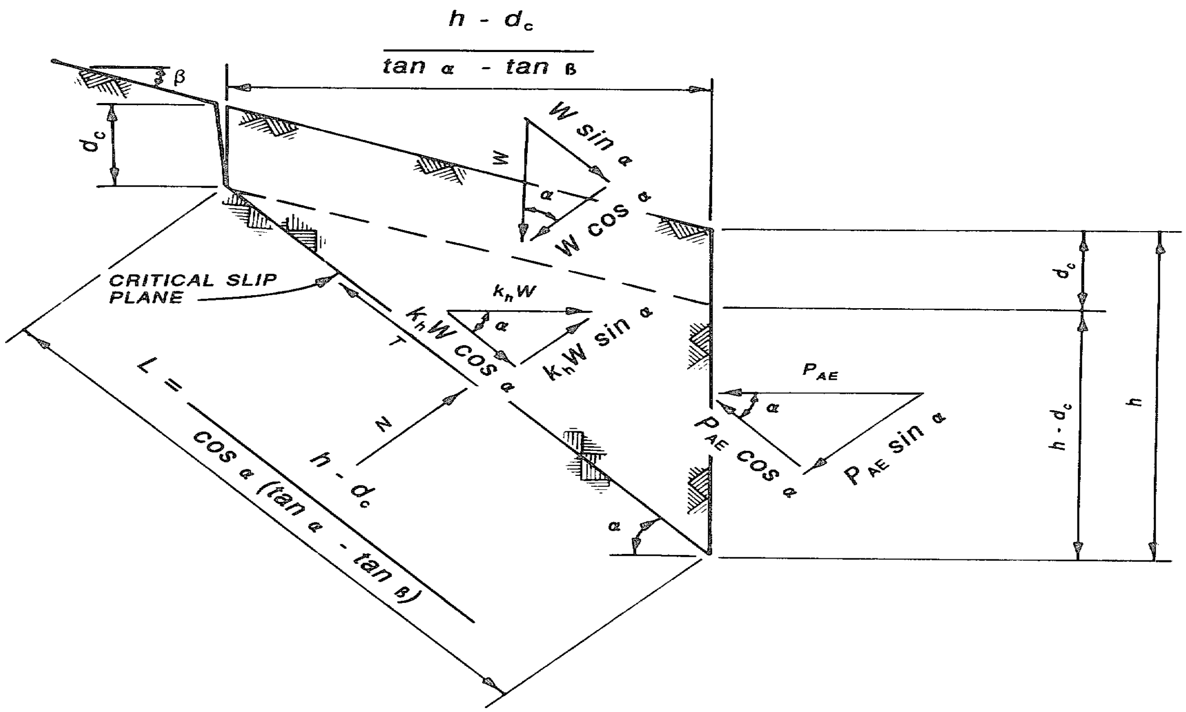
$$c_2 = \frac{\tan\phi(1 + \tan\phi\tan\beta) + \tan\beta + \dfrac{2c(1 + \tan\phi\tan\beta)}{\gamma h} - \dfrac{2V\tan^2\beta(1 + \tan^2\phi)}{\gamma h^2}}{A} \qquad \text{[G-15]}$$

$$\alpha = \tan^{-1}\left(\frac{-c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right) \qquad [G-16]$$

Note: $d_c = 0$, in cohesive soil on resisting side of walls.

G-4. Effect of Water on Critical Slip-Plane Angle. The unit weight of soil (γ) to use in equations for calculating the slip-plane angle should be the average unit weight of soil in the wedge. The average unit weight is determined by using the moist unit weight of soil above the water table and the buoyant unit weight below the water table. The effect of seepage should be taken into account in determining the buoyant unit weight. See Appendices M and N for example calculations.

G-5. Derivation of Equations for Critical Slip-Plane Angle (α) for Driving Wedges for Earthquake Condition. The equations for the critical slip-plane angle (α) for a driving wedge as shown in the figure below for the dynamic case will be derived.



a. Summation of Forces. Summing forces normal to the sliding plane yields

$$N = W \cos \alpha + P_{AE} \sin \alpha - k_h W \sin \alpha$$

The shearing force T is defined by the Mohr-Coulomb failure criterion as

$$T = N \tan \phi + cL = (W \cos \alpha + P_{AE} \sin \alpha - k_h W \sin \alpha) \tan \phi + cL$$

Summing forces parallel to slip plane and inserting the expression for T yields

$$P_{AE} \cos \alpha - k_h W \cos \alpha - W \sin \alpha + (W \cos \alpha + P_{AE} \sin \alpha - k_h W \sin \alpha) \tan \phi + cL = 0$$

$$P_{AE}(\cos \alpha + \tan \phi \sin \alpha) = W(\sin \alpha - \tan \phi \cos \alpha) + k_h W(\cos \alpha + \tan \phi \sin \alpha) - cL$$

Solving for P_{AE} yields

$$P_{AE} = \frac{W[(1 + k_h \tan \phi) \sin \alpha - (\tan \phi - k_h) \cos \alpha] - cL}{\cos \alpha + \tan \phi \sin \alpha}$$

Referring to the figure,

$$W = \frac{\gamma(h^2 - d_c^2)}{2(\tan \alpha - \tan \beta)} \ , \ L = \frac{h - d_c}{\cos \alpha \ (\tan \alpha - \tan \beta)}$$

Inserting the terms for W and L into the equation for P_{AE} yields

$$P_{AE} = \frac{\gamma(h^2 - d_c^2)\left[(1 + k_h \tan \phi) \sin \alpha - (\tan \alpha - k_h) \cos \alpha\right]}{2(\tan \alpha - \tan \beta)(\cos \alpha + \tan \phi \sin \alpha)}$$

$$- \frac{c(h - d_c)}{\cos \alpha(\tan \alpha - \tan \beta)(\cos \alpha + \tan \phi \sin \alpha)}$$

Using the trigonometric relationship $\sec^2 \alpha = \dfrac{1}{\cos^2 \alpha}$

$$P_{AE} = \frac{\gamma\left(h^2 - d_c^2\right)\left[(1 + k_h \tan \phi) \tan \alpha - (\tan \phi - k_h)\right]}{2(\tan \alpha - \tan \beta)(1 + \tan \phi \tan \alpha)}$$

$$- \frac{2c(h - dc) \sec^2 \alpha}{2(\tan \alpha - \tan \beta)(1 + \tan \phi \tan \alpha)}$$

Now using the trigonometric relationship $\sec^2 \alpha = 1 + \tan^2 \alpha$ and rearranging

$$\frac{2P_{AE}}{\gamma\left(h^2 - d_c^2\right)} = \frac{(1 + k_h \tan \phi) \tan \alpha - (\tan \phi - k_h) - \dfrac{2c}{h + d_c}(1 + \tan^2 \alpha)}{\tan \phi \tan^2 \alpha + (1 - \tan \phi \tan \beta) \tan \alpha - \tan \beta} = \frac{m}{n}$$

b. First Derivative of P_{AE} with Respect to α. The first derivative of P_{AE} with respect to α may now be taken, and set equal to zero. The resulting expression is solved for the critical value of α. The first derivative is equal to

$$\frac{d\left[\dfrac{2P_{AE}}{\gamma\left(h^2 - d_c^2\right)}\right]}{d\alpha} = \frac{n\dfrac{dm}{d\alpha} - m\dfrac{dn}{d\alpha}}{n^2}$$

For this derivative to be equal to zero it is only necessary for $\left(n\dfrac{dm}{d\alpha} - \dfrac{dn}{d\alpha}\right)$ to be equal to zero. In order to simplify operations let:

$1 + k_h \tan \phi = a$, $\tan \phi - k_h = b$, and $1 - \tan \phi \tan \beta = d$

Then:

$$m = a \tan \alpha - b - \left(\frac{2c}{h + d_c}\right)(1 + \tan^2 \alpha)$$

$$n = \tan \phi \tan^2 \alpha + d \tan \alpha - \tan \beta$$

$$\frac{dm}{d\alpha} = \left[a - \left(\frac{4c}{h + d_c}\right) \tan \alpha\right] \sec^2 \alpha$$

$$\frac{dn}{d\alpha} = (2 \tan \phi \tan \alpha + d) \sec^2 \alpha$$

$$n \frac{dm}{d\alpha} = \left[a \tan \phi \tan^2 \alpha + ad \tan \alpha - a \tan \beta - \frac{4c}{\gamma(h + d_c)} \tan \phi \tan^3 \alpha \right.$$

$$\left. - \frac{4c}{\gamma(h + d_c)} d \tan^2 \alpha + \frac{4c}{\gamma(h + d_c)} \tan \beta \tan \alpha \right] \sec^2 \alpha$$

$$m \frac{dn}{d\alpha} = \left[2 a \tan \phi \tan^2 \alpha - 2b \tan \alpha \tan \alpha - \frac{4c}{\gamma(h + d_c)} \tan \phi \tan \alpha \right.$$

$$- \frac{4c}{\gamma(h + d_c)} \tan \phi \tan^3 \alpha + ad \tan \alpha - bd - \frac{2c}{\gamma(h + d_c)} d$$

$$\left. - \frac{2c}{\gamma(h + d_c)} d \tan^2 \alpha \right] \sec^2 \alpha$$

$$\frac{m\,\frac{dn}{d\alpha} - n\,\frac{dm}{d\alpha}}{\sec^2\alpha} = \left[-a\tan\phi - \frac{2c}{\gamma(h + d_c)}\,d\right]\tan^2\alpha$$

$$+\ 2b\left[\tan\phi + \frac{4c}{\gamma(h + d_c)}\,(\tan\beta + \tan\phi)\right]\tan\alpha$$

$$+\left[-a\tan\beta + bd + \frac{2c}{\gamma(h + d_c)}\,d\right] = 0$$

Substitute $1 + k_h \tan\phi$ for a , $\tan\phi - k_h$ for b and $1 - \tan\phi\tan\beta$ for d

$$\left[-(1 + k_h\tan\phi)\tan\phi - \frac{2c}{\gamma(h + d_c)}\,(1 - \tan\phi\tan\beta)\right]\tan^2\alpha$$

$$+\left[2(\tan\phi - k_h)\tan\phi + \frac{4c}{\gamma(h + d_c)}\,(\tan\beta + \tan\phi)\right]\tan\alpha$$

$$+\left[-(1 + k_h\tan\phi)\tan\beta + (\tan\phi - k_h)(1 - \tan\phi\tan\beta)\right.$$

$$\left.+\ \frac{2c}{\gamma(h + d_c)}\,(1 - \tan\phi\tan\beta)\right] = 0$$

$$\tan^2\alpha - \left[\frac{2\tan\phi\,(\tan\phi - k_h) + \frac{4c}{\gamma(h + d_c)}\,(\tan\phi + \tan\beta)}{(1 + k_h\tan\phi)\tan\phi + \frac{2c}{\gamma(h + d_c)}\,(1 - \tan\phi\tan\beta)}\right]\tan\alpha$$

$$-\left[\frac{\tan\phi\,(1 - \tan\phi\tan\beta) - (k_h + \tan\beta) + \frac{2c}{\gamma(h + d_c)}\,(1 - \tan\phi\tan\beta)}{(1 + k_h\tan\phi)\tan\phi + \frac{2c}{\gamma(h + d_c)}\,(1 - \tan\phi\tan\beta)}\right] = 0$$

then

$$\alpha = \tan^{-1}\left[\frac{c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right] \qquad [G\text{-}17]$$

where

$$c_1 = \frac{2 \tan \phi\ (\tan \phi - k_h) + \frac{4c}{\gamma(h + d_c)}\ (\tan \phi + \tan \beta)}{(1 + k_h \tan \phi) \tan \phi + \frac{2c}{\gamma(h + d_c)}\ (1 - \tan \phi \tan \beta)} \qquad [G\text{-}18]$$

$$c_2 = \frac{\tan \phi\ (1 - \tan \phi \tan \beta) - (k_h + \tan \beta) + \frac{2c}{\gamma(h + d_c)}\ (1 - \tan \phi \tan \beta)}{(1 + k_h \tan \phi) \tan \phi + \frac{2c}{\gamma(h + d_c)}\ (1 - \tan \phi \tan \beta)} \qquad [G\text{-}19]$$

$$d_c = \frac{c}{(\sin \alpha \cos \alpha - \tan \phi \cos^2 \alpha)\gamma} \qquad [G\text{-}20]$$

G-6. Equations for Critical Slip-Plane Angles for Resisting Wedges for Earthquake Condition. The equations for the critical slip-plane angle (α) for a resisting wedge are derived in the same manner used for the driving wedges. The resulting equations are

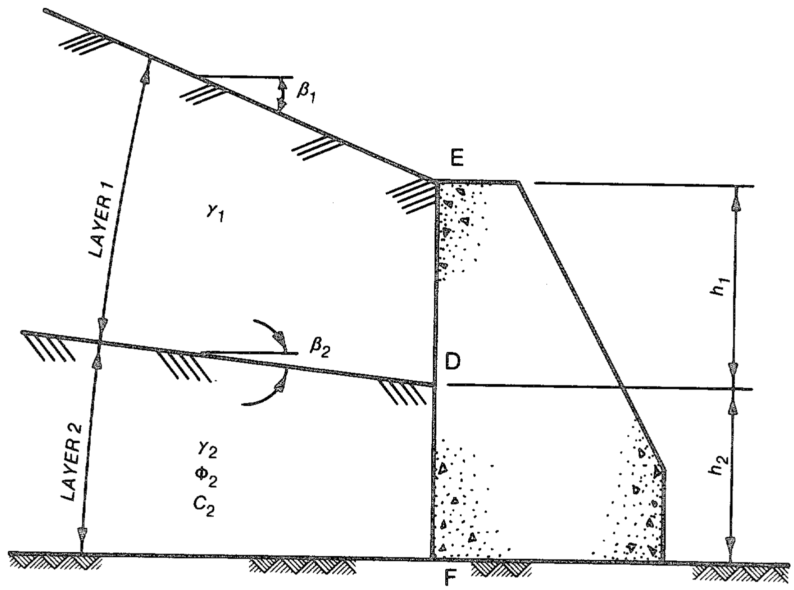
$$\alpha = \tan^{-1}\left[\frac{-c_1 + \sqrt{c_1^2 + 4c_2}}{2}\right] \qquad [G\text{-}21]$$

$$A = (1 + k_h \tan \phi) \tan \phi + \frac{2c(1 + \tan \phi \tan \beta)}{\gamma h} \qquad [G\text{-}22]$$

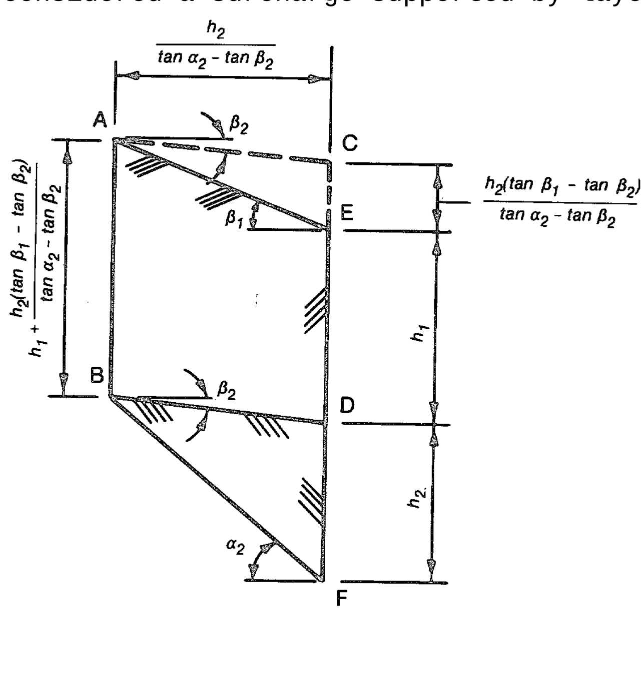
$$c_1 = \frac{2(\tan \phi - k_h) \tan \phi + \frac{4c(\tan \phi - \tan \beta)}{\gamma h}}{A} \qquad [G\text{-}23]$$

$$c_2 = \frac{\tan \phi(1 + \tan \phi \tan \beta) + (\tan \beta - k_h) + \frac{2c(1 + \tan \phi \tan \beta)}{\gamma h}}{A} \qquad [G\text{-}24]$$

G-7. <u>Procedure for Finding the Critical Slip-Plane Angle, for a Wedge in a Layer Below the Top Layer, of a Stratified Backfill</u>. A layered soil system is shown in the figure on the following page.



In order to find the critical slip-plane angle for layer 2, the weight of soil in layer 1 will be considered a surcharge supported by layer 2. See figure below.



The weight of soil in parallelogram ABCD of the figure is a uniformly distributed surcharge that varies with α_2 . The uniformly distributed surcharge does not affect the angle α_2 , but does have the effect of increasing the unit weight of soil in wedge 2. The increased soil unit weight is calculated as follows.

γ' = increased unit weight

$$\frac{\gamma' h_2^2}{2\,(\tan \alpha_2 - \tan \beta_2)} = \frac{\gamma_2 h_2^2}{2\,(\tan \alpha_2 - \tan \beta_2)} + \gamma_1 h_2 \left[\frac{h_1 + \dfrac{h_2\,(\tan \beta_1 - \tan \beta_2)}{\tan \alpha_2 - \tan \beta_2}}{\tan \alpha_2 - \tan \beta_2} \right]$$

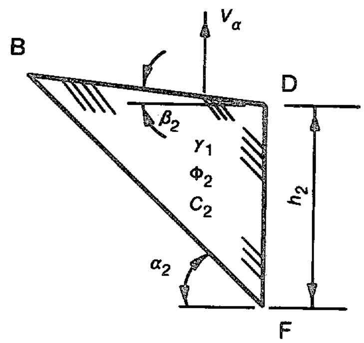
$$\frac{\gamma' h_2^2}{2\,(\tan \alpha_2 - \tan \beta_2)} = \frac{2\gamma_1 h_1 h_2 + \gamma_2 h_2^2}{2\,(\tan \alpha_2 - \tan \beta_2)} + \frac{\gamma_1 h_2^2\,(\tan \beta_1 - \tan \beta_2)}{\left(\tan \alpha_2 - \tan \beta_2\right)^2}$$

$$\gamma' = \frac{2\gamma_1 h_1}{h_2} + \gamma_2 + \frac{2\gamma_1\,(\tan \beta_1 - \tan \beta_2)}{\tan \alpha_2 - \tan \beta_2} \qquad \text{[G-25]}$$

The weight of soil in triangle ACE is considered to be a negative strip surcharge that also varies with α . The value of this strip surcharge is calculated as follows.

$$V_\alpha = - \frac{\gamma_1 h_2^2\,(\tan \beta_1 - \tan \beta_2)}{2\left(\tan \alpha_2 - \tan \beta_2\right)^2} \qquad \text{[G-26]}$$

Using the terms γ' and V_α , we have a wedge whose critical slip-plane angle may be determined from Equations 3-25, 3-28, 3-29, and 3-30. See the figure on the following page.



Equations 3-25, 3-28, 3-29, and 3-30 become:

$$A' = \tan\phi_2 + \frac{2c_2\left(1 + \tan^2\phi_2\right)}{\gamma' h_2} - \frac{2V_\alpha\left(1 + \tan^2\phi_2\right)}{\gamma' h_2^2} \qquad \text{[G-27]}$$

$$c_1' = \frac{2\tan^2\phi_2 + \dfrac{4c_2(\tan\phi_2 + \tan\beta_2)}{\gamma' h_2} - \dfrac{4V_\alpha \tan\beta_2\left(1 + \tan^2\phi_2\right)}{\gamma' h_2^2}}{A'} \qquad \text{[G-28]}$$

$$c_2' = \frac{\tan\phi_2(1 - \tan\phi_2\tan\beta_2) - \tan\beta_2 + \dfrac{2c_2(1 - \tan\phi_2\tan\beta_2)}{\gamma' h_2} + \dfrac{2V_\alpha \tan^2\beta_2\left(1 + \tan^2\phi_2\right)}{\gamma' h_2^2}}{A'} \qquad \text{[G-29]}$$

$$\alpha_2 = \tan^{-1}\left(\frac{c_1' + \sqrt{c_1'^2 + 4c_2'}}{2}\right) \qquad \text{[G-30]}$$

Note that d_c (depth of crack in cohesive soil) is taken as zero in the above equations. This is based upon the assumption that horizontal compressive pressure, due to surcharge, is greater than any negative pressure that might develop due to cohesion.

a. Procedure:

(1) Choose a trial value for α_2 .

(2) Using trial value of α_2, calculate γ' (Equation G-10) and (Equation G-11) V_α.

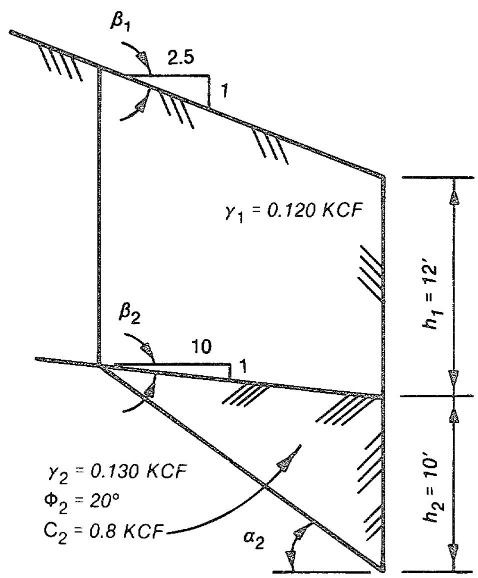
(3) Using the values of γ' and V_α from step 2, calculate α_2 from Equations G-12, G-13, G-14, and G-15.

(4) If α_2 from step 3 is equal to α_2 in step 1 go to step 6. If α_2 from step 3 is not equal to α_2 in step 1 go to step 5.

(5) Use α_2 from step 3 as a new trial value in step 1, and repeat procedure.

(6) Stop. α_2 from step 3 is critical value.

b. Example: Find α_2 critical to nearest 0.5 degree.



From Equation G-10,

$$\gamma' = \frac{2(0.12)(12)}{10} + 0.13 + \frac{2(0.12)(0.4 - 0.1)}{\tan \alpha_2 - 0.1}$$

$$\gamma' = 0.418 + \frac{0.072}{\tan \alpha_2 - 0.1}$$

From Equation G-11,

$$V_\alpha = - \frac{0.12(10)^2(0.4 - 0.1)}{2\left(\tan \alpha_2 - 0.1\right)^2} = - \frac{1.80}{\left(\tan \alpha_2 - 0.1\right)^2}$$

(1) $\underline{\alpha_2 \text{ for long term loading } (\phi_2 = 20° , c_2 = 0)}$

(a) <u>First trial</u>: $\alpha_2 = 45°$, $\gamma' = 0.4980$, $V_\alpha = -2.2222$

Using Equations G-12, G-13, G-14, and G-15,

$$A' = 0.36398 - \frac{2(-2.2222)(1.132474)}{0.4980(10)^2} = 0.465038$$

$$c'_1 = \frac{0.264948 - \dfrac{4(-2.2222)(0.1)(1.132474)}{0.4980(10)^2}}{0.465038} = 0.613200$$

$$c'_2 = \frac{0.36397(1 - 0.36397 \times 0.1) - 0.1 + \dfrac{2(-2.2222)(0.01)(1.132474)}{0.4980(10)^2}}{0.465038}$$

$$c'_2 = 0.536971 , \quad \alpha_2 = 47.75° \quad 48° \neq 45°$$

(b) <u>Second trial</u>: $\alpha_2 = 48.0°$, $\gamma' = 0.4892$, $V_\alpha = -1.7624$

$$A' = 0.36397 - \frac{0.022649(-1.7624)}{0.4892} = 0.445566$$

$$c'_1 = \frac{0.264948 - \dfrac{0.004530(-1.7684)}{0.4892}}{0.445566} = 0.631260$$

$$c'_2 = \frac{0.250723 + \dfrac{0.0002265(-1.7624)}{0.4892}}{0.445566} = 0.560875$$

$$\alpha_2 = 48.45^\circ \quad 48.5^\circ \neq 48^\circ$$

(c) <u>Third trial</u>: $\alpha_2 = 48.5^\circ$, $\gamma' = 0.4879$, $V_\alpha = 1.6957$

$$A' = 0.36397 - \frac{0.022649(-1.6957)}{0.4879} = 0.442687$$

$$c'_1 = \frac{0.264948 - \dfrac{0.004530(-1.6957)}{0.4879}}{0.442687} = 0.634064$$

$$c'_2 = \frac{0.250723 + \dfrac{0.0002265(-1.6957)}{0.4879}}{0.442687} = 0.564588$$

$$\alpha_2 = 48.56^\circ \quad 48.5^\circ$$

$$\underline{\underline{\alpha_2 = 48.5^\circ}}$$

(2) <u>α_2 for short term loading ($\phi_2 = 0$, $c_2 = 0.8$ ksf)</u>

(a) <u>First trial</u>: $\alpha_2 = 45^\circ$, $\gamma' = 0.4980$, $V_\alpha = -2.2222$

$$A' = \frac{2(0.8)}{0.498(10)} - \frac{2(-2.2222)}{0.498(10)^2} = 0.410530$$

$$c'_1 = 2 \tan \beta_2 = 0.2$$

$$c'_2 = \frac{-0.1 + \dfrac{2(0.8)}{0.498(10)} + \dfrac{2(-2.2222)(0.01)}{0.498(10)^2}}{0.410530} = 0.537936$$

$$\alpha_2 = 40.04^\circ \approx 40^\circ$$

(b) <u>Second trial</u>: $\alpha_2 = 40°$, $\gamma' = 0.5154$, $V_\alpha = -3.2951$

$$A' = \frac{0.16}{0.5154} - \frac{0.02(-3.7218)}{0.5154} = 0.438304$$

$$c'_1 = 0.2 \ , \ c'_2 = \frac{-0.1 + \frac{0.16}{0.5154} + \frac{0.0002(-3.2951)}{0.5154}}{0.438304} = 0.477203$$

$$\alpha_2 = 38.59° \approx 38.5° \neq 40°$$

(c) <u>Third trial</u>: $\alpha_2 = 38.5°$, $\gamma' = 0.5215$, $V_\alpha = -3.7218$

$$A' = \frac{0.16 - 0.02(-3.7218)}{0.5215} = 0.449542$$

$$c'_1 = 0.2 \ , \ c'_2 = \frac{-0.1 + \frac{0.16 + 0.0002(-3.7218)}{0.5215}}{0.449542} = 0.456865$$

$$\alpha_2 = 38.07° \approx 38° \neq 38.5°$$

(d) <u>Fourth trial</u>: $\alpha_2 = 38°$, $\gamma' = 0.5237$, $V_\alpha = -3.8781$

$$A' = \frac{0.16 - 0.02(-3.8781)}{0.5237} = 0.453622$$

$$c'_1 = 0.2 \ , \ c'_2 = \frac{-0.1 + \frac{0.16 + 0.0002(-3.8781)}{0.5237}}{0.453622} = 0.449796$$

$$\alpha_2 = 37.89° \quad 38°$$

$$\underline{\underline{\alpha_2 = 38°}}$$

(3) Check α_2 = 48.5° (for long term loading) using Equation 3-23

$$W = \frac{0.13(10)^2}{2\ (\tan\alpha_2 - 0.1)} + \frac{0.12(12)(10)}{\tan\alpha_2 - 0.1} + \frac{0.12(10)^2\ (\tan\beta_1 - \tan\beta_2)}{2\left(\tan\alpha_2 - 0.1\right)^2}$$

$$W = \frac{20.9}{\tan\alpha_2 - 0.1} + \frac{1.8}{\left(\tan\alpha_2 - 0.1\right)^2}$$

$$P_{EE} = \frac{W\ (\tan\alpha_2 - \tan\phi_2)}{1 + \tan\phi_2\ \tan\alpha_2}$$

α_2 = 47.5°: W = 22.9149 kips

$$P_{EE} = \frac{22.9149(1.091309 - 0.36397)}{1 + 0.36397(1.091309)} = 11.9288 \text{ kips}$$

α_2 = 48.5°: W = 21.9812 kips

$$P_{EE} = \frac{21.9812(1.130294 - 0.36397)}{1 + 0.36397(1.130294)} = 11.9348 \text{ kips} > 11.9288 \text{ kips}$$

α_2 = 49.5°: W = 21.0869 kips

$$P_{EE} = \frac{21.0869(1.170850 - 0.36397)}{1 + 0.36397(1.170850)} = 11.9304 \text{ kips} < 11.9348$$

The critical α_2 equals 48.5° which checks with the value computed in paragraph G-7b(1)(c).

Check α_2 = 38° (for short term loading) using Equation 3-23

$$W = \frac{20.9}{\tan \alpha_2 - 0.1} + \frac{1.8}{\left(\tan \alpha_2 - 0.1\right)^{2'}} \quad L = \frac{10}{\cos \alpha_2 (\tan \alpha_2 - 0.1)}$$

$$P_{EE} = W \tan \alpha_2 - c_2 L/\cos \alpha$$

$\alpha_2 = 37°$: $W = 36.1931$ kips , $L = 19.1589$ ft

$$P_{EE} = 36.1931(0.753554) - \frac{0.8(19.1589)}{0.798636} = 8.0818 \text{ kips}$$

$\alpha_2 = 38°$: $W = 34.5554$ kips , $L = 18.6268$ ft

$$P_{EE} = 34.5554(0.781286) - \frac{0.8(18.6268)}{0.788011} = 8.0875 \text{ kips} > 0.0818$$

$\alpha_2 = 39°$: $W = 33.0185$ kips , $L = 18.1289$ ft

$$P_{EE} = 33.0185(0.809784) - \frac{0.8(18.1289)}{0.777146} = 8.0758 \text{ kips} < 8.0875$$

The critical α_2 equals 38° which checks with the value computed in paragraph G-7b(2)(d)